# **Thermophysical Properties of Interfacial Layer in Nanofluids**

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Although recent experiments have revealed that nanofluids have superior thermal conductivities to base fluids, the inherent physics are not fully understood. In this study, an interfacial layer, competing with Brownian motion as a corresponding mechanism, is conceptually connected with the surface-charge-induced electrical double layer. By applying colloidal science, the first explicit equations for the thickness and thermal conductivity of the layer are obtained. A fractal model including the new concept of the layer is developed. The model predictions are compared with experimental data for effects of pH, temperature, volume fraction, and primary particle size of CuO–water nanofluids.

#### 1. Introduction

Nanofluid, a colloidal liquid with nanoparticles, has increasingly attracted worldwide attention, since 1995 when Choi<sup>1</sup> introduced a new kind of the heat-transport fluid. Many experimental results have followed the unbelievable report<sup>2</sup> of Choi and his colleagues in 2001, citing more than 150% increase in effective thermal conductivity  $K_{\rm eff}$  of carbon nanotube-olefin oil nanofluid with respect to that of the base fluid  $K_{\rm f}$ . The mainstream experimental research was to find the best combination of particles and solvents<sup>3-17</sup> and to find the most significant systematic parameters as well. The parameters may be sorted into three classes, i.e., particle-related, fluid-related, and interrelated parameters. The first kind includes size,<sup>3-5</sup> morphology (elongated,<sup>2,6</sup> spherical,<sup>3,7,8</sup> and mass fractal-like<sup>9-14</sup> shapes), and volume fraction of particles. The second would be thermophysical properties of the base fluid such as viscosity, thermal conductivity, and temperature.<sup>3,10,15</sup> Interface chemical effect or interaction between the particles and base fluid<sup>6,14</sup> would correspond to the last class.

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Besides great interest in the  $K_{\rm eff}$  of the nanofluid, several groups initiated investigation of more practical application fields such as microchannel heat sink,<sup>18</sup> forced convective heat transfer fluids,<sup>19</sup> and a lubricating fluid with high thermal conductivity.<sup>20</sup> Although nanofluids in the field certainly look promising, it is not rare to find large discrepancies among the experimental data obtained by different research groups. Keblinski et al.<sup>21</sup> addressed this as one of the primary obstacles to the development of nanofluids. Also, they concluded that poor characterization of the suspension, e.g., quantifying how stable the colloid is, could explain the inconsistencies in experiments in terms of inconsistent initial conditions of the nanofluids. Recently, we demonstrated that pH of the CuO-water nanofluid controlled the surface charge states as well as the interaction potentials. By using the Derjaguin-Landau-Verwey-Overbeek (DLVO) linearized mean field theory,<sup>22</sup> we successfully quantified the suspension stability of the nanofluid and found that the  $K_{\rm eff}$  was strongly correlated with the surface charge states.<sup>14</sup> Right after this study, Prasher et al.<sup>23</sup> suggested that the  $K_{\rm eff}$  could be greatly changed by aggregation kinetics and the characteristic time for aggregation was influenced by various systematic parameters. Therefore, it is necessary to know a complete set of the three classes of parameters for a model evaluation or even for a fair comparison between experiments.

Keblinski et al.<sup>24</sup> who gave the first reasonable insights of the mechanisms showed that Brownian motion (BM) was not responsible for the enhancement of the  $K_{\text{eff}}$ . However, a few groups such as Jang and Choi,<sup>25</sup> Kumar et al.,<sup>26</sup> and Prasher et al.<sup>27</sup> are still arguing that the motion-related effect plays a dominant role, though their predictions were made with

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<sup>\*</sup> Author to whom correspondence should be addressed. Electronic mail: donglee@pusan.ac.kr. Tel +82-51-510-2365. Fax +82-51-512-5236. (1) Choi, S. U. S. ASME FED **1995**, 231, 99.

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different equations. It is notable that the groups have all disregarded that the metal oxide particles used for the model evaluation were highly aggregated (see ref 16 in Jang and Choi,<sup>25</sup> ref 14 in Kumar et al., <sup>26</sup> and refs 1 and 4 in Prasher et al.<sup>27</sup>). Since aggregation of isolated (primary) particles decreases the speed at which particles are moving, use of the primary particle size can considerably overestimate the effect of the BM. Recent molecular dynamic simulation<sup>28</sup> reveals that the hydrodynamic BM mechanism has only a minor effect on the  $K_{eff}$  even for a relatively high dose of nanoparticles (ca. 3.3 vol %). In this regard, the BM effect is not taken into account in this study.

On the other hand, some people have proposed a hypothetical concept of the ordered liquid layering around the particle-liquid interface as another reason for the enhancement of the  $K_{\rm eff}$ .<sup>17,24,29–31</sup> The good predictions with relevant models were, however, based on arbitrarily assumed values for thickness and thermal conductivity of the interfacial layer. Under the assumption of constant properties of the layer, any versions of the relevant model have never explained the pH dependence of the  $K_{\rm eff}^{14}$  or the influence of the surface modification.<sup>3,6</sup> The last mechanism suggested by Keblinski et al.<sup>24</sup> is a percolation behavior of heat transport especially when the colloidal particles are agglomerated. The heat is preferably transferred through the aggregate backbone in a liquid sphere,<sup>23</sup> so that the nanofluid containing aggregates may have higher effective thermal conductivity, compared to the case of disassembling the aggregates into their primary particles.

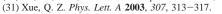
In this study, the author attempts to connect the concept of an interfacial layer with an electrical double layer (EDL) forming around the surfaces of particles in suspension. The thickness and thermal conductivity of the interfacial layer thereby have functional dependencies on systematic parameters. Regarding this, the author proposes a mathematical fractal model incorporating the afore-mentioned parameters and mechanisms. Quantitative comparisons of the model with the various experimental data will be seen.

#### 2. Experimental Section

The experimental procedures from the sample preparation to the characterization of the colloid<sup>14</sup> are revisited in view of a full set of data and briefly addressed as follows. As-received CuO nanoparticles (Aldrich, cat. no 54486-8) are highly agglomerated. Right after the particles are immersed into deionized water, the colloid is strongly agitated with 700 W ultrasonic waves. Such a high-powered ultrasonic treatment is thought in general to improve the stability of the colloids by tearing the agglomerates into their primary particles. But, many tests for different durations and powers of ultrasonication (Jeiotech, Kr., ULH-700S) revealed that the particles sustain their fractal shapes (see Figure 1 in ref 14).

A photon correlation method (Otsuka Electronics, Jp., ELS-8000) was used to estimate hydrodynamic radius  $R_{\rm h}$  of particles moving in liquid<sup>14,22</sup> through the measurement of diffusion coefficient of the particles. As far as Brownian motion of aggregates concerns, the  $R_{\rm h}$ of aggregates is more important than the radius of their primary particles a. It is surprising that most of researchers claiming the BM mechanism have not attempted to measure the  $R_{\rm h}$ . Figure 1shows that the  $R_h$  of CuO aggregates in water ranges from 90 nm to 140 nm, depending on the pH of the nanofluid, whereas a was estimated to about 17.5 nm. Such large aggregates seem to be too big to generate any apparent size effect.25

The pH of the colloids was altered by adding suitable amounts of HCl or NaOH (Aldrich, cat. no 31894-9 or 30657-6, respectively). NaCl (Aldrich, cat. no 20443-9) as an electrolyte was used for keeping the total ionic strength consistent, but the dose



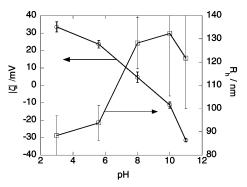


Figure 1. Effect of the pH of a CuO-water nanofluid on the suspension stability: the measurements are made at T = 25 °C and  $\phi_{exp} = 0.003$ . The error bars represent standard deviations of data from more than 10 measurements.

level was much lower than the critical value (ca.  $5 \times 10^{-2}$  mol dm<sup>-3</sup>) at which particles are settled by a violent aggregation. The volume fraction of particles is kept constant at 0.3% during the experiments. Effective thermal conductivity  $(K_{eff})$  of the fluid is measured at 25 °C by a conventional transient hot wire device. The zeta potential  $\zeta$  of the nanofluid measured from electrokinetic data (Otsuka Electronics, Jp., ELS-8000) was combined with a surface complexation model to obtain surface charge states. More details of the experimental and theoretical procedures are explained elsewhere.<sup>14</sup> The author notes that these data are pretty close to the complete data set which was mentioned in the Introduction. The whole sets of the data at different pHs will be basically used for evaluating the present model. As such data have not been reported for other materials, further evaluation of the model will be limited to the CuO-water nanofluid in the literature. The model predictions for the influences of volume fraction, temperature, and primary particle size will also be presented.

## 3. Thermophysical Properties of Surface **Charge-Induced Interfacial Layer**

A. Thickness of Interfacial Layer (t). When materials such as oxides, sulfides, and insoluble salts are immersed in aqueous solution, they acquire surface charges, the presence of which builds up the surface electrostatic potentials. The surface charges attract and bind counterions with opposite polarity in liquid. Regarding that the ions are often hydrated, one may postulate that association of such ions with water molecules at the interface is the origin of the hypothetical charge-induced interfacial layer in the literature. The alternating alignment of the (often hydrated) ions may form liquid columns as depicted in Figure 2 with a bold-dotted box. The strong ionic bonds between the surface and the columns seem likely to reduce the Kapitza resistance significantly and make arrangements of the ions mimic the crystal structure of solid atoms at the surface as well. More detail mechanism about the EDL is described elsewhere.14,22

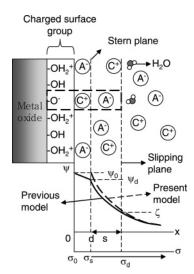
Since the thickness of the EDL often scales as a reciprocal Debye-Huckel parameter  $\kappa^{-1} (= \sqrt{\epsilon_0 \epsilon_r R_0 T/2000 F^2 I})^{14,22}$  the thickness t can be expressed as

$$t = C\kappa^{-1} \tag{1}$$

where  $\epsilon_0$  is vacuum permittivity, F is Faraday constant, and  $R_0$ is gas constant. The remaining properties such as dielectric constant  $\epsilon_r$ , ionic strength *I*, and temperature *T* are all related to the solution. Thus, the thickness of the interfacial layer has now base-fluid dependence. In particular, increasing the temperature makes the layer thicker, resulting in an increase of the apparent volume fraction, through which the  $K_{\rm eff}$  of nanofluids can then

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**Figure 2.** Structure of electrical double layer (EDL) formed at the metal oxide—water interface:<sup>14,22</sup> dehydrated counterion A<sup>-</sup> or C<sup>+</sup> bonded strongly to the charged group at surface forms an immobile part of the EDL (stern layer). In contrast, in the outer region named as a diffuse layer, such ions are bound weakly, resulting in gradual decrease in the potential. The  $\sigma_0$ ,  $\sigma_s$ , and  $\sigma_d$  are charge densities at the surface (0 plane), in the stern plane, and in the diffuse layer, respectively.

be enhanced. The temperature dependence which has been thought to be a justification of the BM models<sup>25-27</sup> can also be explained by the concept of the interfacial layer.<sup>17,24,29-31</sup>

In the present condition, pH of solution can alter the  $\kappa^{-1}$  through the ionic strength *I*. For example, when the pH increases from 3 to 8, keeping the other parameters constant, the  $\kappa^{-1}$  increases from 7 to 13 nm. Taking  $\kappa^{-1}$  as the layer thickness *t* (*C* = 1), the *t* value is comparable to the primary particle radius of 17.5 nm but negligible compared to the *R*<sub>h</sub> (from 90 to 140 nm) of the aggregates. As the primary particles get smaller, the contribution of the layer become greater.

**B.** Thermal Conductivity of Interfacial Layer (K<sub>l</sub>). When recounting the concept of "liquid column" described in the previous section, if there exist more sites to launch the column (total charged surface density  $\Gamma_{ion}$ ) and more ions to strengthen the column structure (ion density in the EDL given by  $\sigma_{d\kappa}$ , i.e., the product of charge density in the diffusive plane  $\sigma_d$  and the Debye-Huckel parameter  $\kappa$ ),<sup>14</sup> this would in turn facilitate phonon transport from particle to liquid.<sup>32</sup> Interestingly, the experiment revealed that  $K_{\rm eff}$  was expressed well as a power of  $\Gamma_{\rm ion}\sigma_{\rm d}\kappa$ .<sup>14</sup> Hence, it seems plausible to assume that the thermal conductivity of the layer  $K_1$  is also expressed as a power function of  $\Gamma_{ion}\sigma_d\kappa$ . Note that the  $K_1$  of the EDL is limited by two extremes, i.e.,  $K_f$ and  $K_s$  (thermal conductivity of the CuO). The upper limit ( $K_1$  $= K_{\rm s}$ ) would be the case at the condition for complete ionization of the surface ( $\Gamma_{ion} = \Gamma_{tot} = 5.88 \times 10^{-6} \, \text{mol m}^{-2}$ ) and maximizing  $\sigma_{d\kappa}$  (at pH = 3). In this way, one may derive a reasonable functional relationship between  $K_1$  and  $\Gamma_{ion}\sigma_d\kappa$  as

$$\frac{K_{\rm l} - K_{\rm f}}{K_{\rm s} - K_{\rm f}} = \left[\frac{(\Gamma_{\rm ion}\sigma_{\rm d}\kappa)_{\rm pH}}{\Gamma_{\rm tot}(\sigma_{\rm d}\kappa)_{\rm pH=3}}\right]^{\alpha}$$
(2)

The exponent  $\alpha$  is an adjustable parameter for fitting the experimental data. However, once  $\alpha$  has been determined, the value was a fixed constant throughout all predictions. In eq 2, the  $K_1$  value has an implicit temperature dependence through the

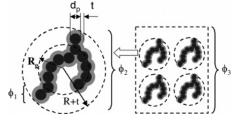


Figure 3. Geometrical configuration of fractal aggregates covered with interfacial layer in a nanofluid.

terms of  $\Gamma_{ion}$ ,  $\sigma_d$ , and  $\kappa$ . The validity of the equation will be discussed in the Discussion. In the Appendix, a theoretical procedure to estimate the surface charge states is described in detail, which is modified to consider varying temperatures on the basis of the previous theory.<sup>14</sup>

# 4. Fractal Model

The fractal feature of aggregates is generally described by the following power law.<sup>33–35</sup> The number of primary particles  $N_p$  within an aggregate scales as the outer radius R or the radius of gyration  $R_g$  with respect to the radius of primary particles a

$$N_{\rm p} = k_0 \left(\frac{R}{a}\right)^{D_{\rm f}} = k_{\rm g} \left(\frac{R_{\rm g}}{a}\right)^{D_{\rm f}} \tag{3}$$

where  $D_{\rm f}$  is a fractal dimension, and  $k_0$  and  $k_{\rm g}$  are prefactors based on *R* and  $R_{\rm g}$ , respectively. Koylu et al.<sup>33</sup> performed a computer simulation for a diffusion-limited cluster-cluster aggregation and concluded that  $k_0$  is related to  $k_{\rm g}$  as a function of  $D_{\rm f}$  as

$$k_0 = k_{\rm g} \left( \frac{D_{\rm f}}{D_{\rm f} + 2} \right)^{D_{f/2}} \tag{4}$$

The  $D_{\rm f}$  value can be estimated from the slope of a log-log plot of  $N_{\rm p}$  against R/a according to eq 3 and ranges typically from 1.5 to 1.8 for the present shape of aggregates.<sup>13,17,33-35</sup> Also, the  $k_0$  obtained from the *y*-intercept in the plot is typically close to unity. The values of  $D_{\rm f} = 1.6$  and  $k_0 = 1.34$  taken for the best fit in this study are comparable to those of Kim and Yuan,<sup>35</sup>  $D_{\rm f}$ = 1.67 and  $k_0 = 0.83$ ; Wang et al.,<sup>17</sup>  $D_{\rm f} = 1.57$  and  $k_0 = 1.0$ ; Xuan et al.,<sup>13</sup>  $D_{\rm f} = 1.51$  and  $k_0 = 1.07$ ; and Lee and Choi,<sup>34</sup>  $D_{\rm f}$ = 1.7 and  $k_0 = 1.34$ . Substituting eq 4 to eq 3 gives

$$R_{\rm g} = R \left( \frac{D_{\rm f}}{D_{\rm f} + 2} \right)^{1/2} \tag{5}$$

While drying the sampled nanofluid on a grid for TEM observation, aggregates are moved and sometimes overlapped on the grid, which often makes it difficult to measure *R* precisely, whereas light scattering can directly measure  $R_h$ ,<sup>14,22</sup> which approximates  $R_g$  within 13% error.<sup>35</sup> *R* and  $N_p$  are then obtained with given values of  $R_g$ ,  $D_f$ , and  $d_p$  by eqs 3 and 5, respectively. Those are delivered to the following model for prediction of  $K_{eff}$ .

Similarly to the treatment of an aggregate presented by Wang et al.,<sup>17</sup> the present model consists of three sequential steps, from a primary particle with the EDL to the whole system of nanofluid via an equivalent sphere including an aggregate. Figure 3 shows

<sup>(32)</sup> Xue, L.; Keblinski, P.; Phillpot, S. R.; Choi, S. U. S.; Eastman, J. A. J. Chem. Phys. 2003, 118, 337–339.

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a geometrical configuration of the aggregates covered with the EDL having a thickness t in nanofluid.

Step 1. Primary Particle Covered with Interfacial Layer. As a first step, a primary sphere covered with a layer is treated as an imaginary sphere with radius of a + t. The volume fraction  $\phi_1$  of the core solid sphere in the core-shell structure is given by

$$\phi_1 = \left(\frac{a}{a+t}\right)^3 \tag{6}$$

As pH goes far from the point of zero charge (PZC)<sup>14</sup> of CuO in water, with a decrease in *t*, the  $\phi_1$  value increases up to 14.4%. The Bruggeman effective medium theory (EMT), which is applicable to the whole range of concentration of an inclusion,<sup>17</sup> is used to predict the effective thermal conductivity  $K_{c^1}$  of the core—shell sphere by

$$\phi_1 \left[ \frac{K_{\rm s} - K_{\rm c^1}}{K_{\rm s} + 2K_{\rm c^1}} \right] + (1 - \phi_1) \left[ \frac{K_{\rm l} - K_{\rm c^1}}{K_{\rm l} + 2K_{\rm c^1}} \right] = 0 \tag{7}$$

where  $K_s$  and  $K_1$  have the same meanings as in eq 2. An explicit expression of  $K_{c^1}$  is obtained by solving eq 7 for  $K_{c^1}$  as

$$K_{\rm c1} = \frac{1}{4} [3\phi_{\rm l}(K_{\rm s} - K_{\rm l}) + (2K_{\rm l} - K_{\rm s}) + \sqrt{A}]$$
(8)

where the term *A* is given by  $A = [3\phi_1(K_s - K_l) + (2K_l - K_s)]^2 + 8K_sK_l$ .

Step 2. An Aggregate Consisting of the Core–Shell Primary Particles. An aggregate with an interfacial layer is modeled as a solid-phase element of another imaginary sphere with radius of R + t (see Figure 3). A volume fraction  $\phi_2$  of the liquidlayered aggregate in the imaginary sphere is accordingly given by

$$\phi_2 = \frac{(a+t)^3 N_{\rm p}}{(R+t)^3} \tag{9}$$

The Bruggeman model is again used to yield the thermal conductivity  $K_c^2$  of the second imaginary sphere containing the aggregate, layer, and liquid. As the liquid-layered aggregate becomes a new solid fraction in the model,  $K_s$  and  $K_c^1$  in eq 7 are replaced by  $K_c^1$  and  $K_c^2$ , respectively

$$\phi_2 \left[ \frac{K_{c^1} - K_{c^2}}{K_{c^1} + 2K_{c^2}} \right] + (1 - \phi_2) \left[ \frac{K_{f} - K_{c^2} K_{c^2}}{K_{f} + 2K_{c^2}} \right] = 0 \quad (10)$$

Step 3. The Whole System where the Second Imaginary Spheres Are Suspended. As number concentration (*n*) of the imaginary spheres should be identical to that of the original aggregates, it is determined by dividing the original volume fraction ( $\phi_{exp}$ ) of particles by mean volume ( $v_a$ ) of solid-phase aggregates as

$$n = \frac{\phi_{\exp}}{v_a} = \frac{3\phi_{\exp}}{N_p 4\pi a^3} \tag{11}$$

The product of *n* and mean volume  $v_s$  of the imaginary spheres gives the volume fraction  $\phi_3$  of the spheres in liquid by

$$\phi_3 = nv_s = n\frac{4\pi}{3}(R+t)^3$$
(12)

Assuming that the aggregates are uniformly distributed in liquid, the MG model<sup>17</sup> is finally used to predict the  $K_{\text{eff}}$  of the nanofluid as

$$\frac{K_{\rm eff}}{K_{\rm f}} = \frac{(1-\phi_3)(K_{\rm c^2}+2K_{\rm f})+3\phi_3K_{\rm c^2}}{(1-\phi_3)(K_{\rm c^2}+2K_{\rm f})+3\phi_3K_{\rm f}}$$
(13)

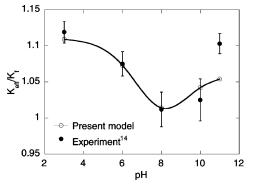
# 5. Results

A. A Contribution of Brownian Motion of Fractal Aggregates. Again in Figure 1, as pH departs from the isoelectric point of CuO particles (IEP  $\approx 8.5$ ), the  $\zeta$  potential increases gradually along with an increase of surface potential ( $\psi_0$ ). Since electrostatic repulsion energy between particles is proportional to  $\zeta^2$  through DLVO theory,<sup>14,22</sup> the the colloidal system becomes less stable at this time. In this way, particles in liquid get bigger at an unstable pH condition, as seen in Figure 1. The  $R_h$  varies by about 48% at most, depending on the pHs. However, the BM model of Kumar et al.<sup>26</sup> for  $R_h$  shows that  $K_{eff}$  varies by only 1% or less at the moment.

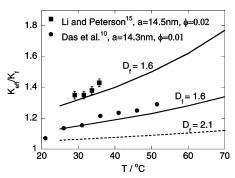
B. Model Prediction for pH Dependence of Keff. Many ions, that exist in the region of  $x > \kappa^{-1}$ , can be still associated with the afore-mentioned liquid columns, thereby decreasing the potential in the region. Here, the effective thickness of the interface layer is first approximated to double the  $\kappa^{-1}$  (C = 2 in eq 1). The value of C is kept constant throughout the predictions in this study. The systematic parameters for the experiment and physical constants used for the model prediction are listed in Table 1. Figure 4 shows that the increment of the effective thermal conductivity  $(K_{eff} - K_f)$  increases by a factor of 8 as pH decreases from 8 to 3 and the present model successfully predicts the pH dependence. To gain an insight into the pH dependence, any changes in all parameters at various pHs are monitored in Table 2. While decreasing the pH from 8 to 5.6, surface charge states in terms of ionized site density ( $\Gamma_{ion}$ ) and charge density ( $\sigma_d$ ) are both increased dramatically, but along with the very little increase in  $\phi_1$ . The thermal conductivity of the EDL ( $K_1$ ) is therefore enhanced by a factor of 2. This in turn increases the  $K_{c^1}$  as well as  $K_{c^2}$  by almost a factor of 2, subsequently. At this time, the outer radius of equivalent spheres containing aggregates (R +t) decreases by only 24%. Thus, the volume fraction of the equivalent spheres  $\phi_3$  decreases by 27%. In comparison to the changes in the thermal conductivities  $K_1$ ,  $K_c^1$ , and  $K_c^2$ , decreasing volume fraction  $\phi_3$  generates a minor adverse effect on the  $K_{\text{eff}}$ . So, it is clear that the abnormal enhancement of the  $K_{eff}$  is primarily due to the large increase in the thermal conductivity of the interfacial layer, which might be interpreted as the enhanced phonon transport as inferred before.

**C. Prediction of Effects of Temperature, Volume Fraction, and Primary Particle Size.** In this section, the present model is further evaluated by comparing the predictions with other experimental results on the effects of temperature, volume fraction, and primary particle size. As indicated at the end of Introduction, surface charge states and fractal information ( $D_f$ and  $R_h$ ) should all be predetermined to give a correct prediction. But, such complete data sets for typical nanofluids have never been reported before. Since the charge states are the fundamental parameters determining  $D_f$  and  $R_h$ , if the pH values of the colloidal liquids with the same contents are kept constant, the fractal information is likely not varied much. In this regard, the comparisons are limited to the case of CuO–water nanofluids. The data sets at pH = 7 in the previous section are used again for the predictions in this section.

The temperature effect is first chosen, because it has been considered evidence for the BM mechanism. There are two groups<sup>10,15</sup> who experimentally report the temperature effect for CuO–water nanofluids at two different volume loadings of particles. As equilibrium constants ( $K_p$  and  $K_d$ ) for the surface ionization reaction are very sensitive to the temperature



**Figure 4.** Model prediction of the pH dependence of the effective thermal conductivity of the CuO–water nanofluid; the experiments are made at a volume fraction of 0.003.



**Figure 5.** Comparison of the present model with experimental data for CuO–water nanofluid at different temperatures. The solid lines represent the model predictions at the conditions of the two groups, and the dotted line is produced by the model with  $D_f = 2.1$  for the condition of Das et al.

*T* (see eq A3), the surface charge states in terms of  $\Gamma_{\text{ion}}$  and  $\sigma_{\text{d}}$  rapidly increases with *T*, as seen in Table 3, whereas, the thickness  $\kappa^{-1}$  slowly increases with *T*. Thus, the thermal conductivity of the EDL ( $K_1$  in eq 2) has a strong temperature dependence.

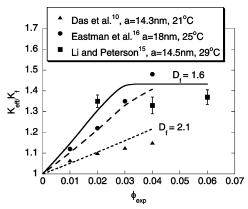
As expected, Figure 5 shows that the present interfacial model predicts such a dependence. Also, note that the predictions for two different doses of CuO particles to water are in good agreements with the experiments, which is achieved with no further assumption. To know the effect of particle morphology, i.e.,  $D_f$ , the prediction for  $D_f = 2.1$  is also seen with a dotted line in the figure. At high  $D_f$ , in the denser structure of aggregates, the EDL is more often overlapped. This decreases the effective volume fraction  $\phi_3$  faster than increasing the effective thermal conductivity  $K_c^2$ . Thus, at high  $D_f$ , the temperature effect is not as great as at low  $D_f$ , which means that open-structured aggregates in suspension is more desirable for enhancing the heat transport. This is consistent with the concept of "clustering" proposed by Keblinski et al.<sup>24</sup>

Second, the effect of volume fraction  $\phi_{exp}$  on the  $K_{eff}$  is investigated. The literature data show three distinct differences in the effect, which are highlighted in Figure 6. In Das et al.,<sup>10</sup> the nanofluid with the smallest primary particles has the smallest  $K_{eff}$  compared to the others. This seems to contradict the conventional size effect. The  $K_{eff}$  of the nanofluid of Eastman et al.<sup>16</sup> linearly increases with particle loadings, while that of Li and Peterson<sup>15</sup> has a flattened  $\phi$  dependence. Why are there such distinct differences even for nanofluids of the same composition? Keblinski et al.<sup>21</sup> recently addressed this, i.e., the lack of agreement between results obtained by different groups, as one of the big obstacles for development. This means that experimentation itself is a challenge. Are these all experimental errors?

Table 1. Parameters Used in the Present Study

experiment	pH = 3-11; $C_{\text{NaCl}} = 5 \times 10^{-4} \text{ mol dm}^{-3}$ ; $T = 298 \text{ K}$ ;
	$K_{\rm f}{}^a = 0.613 \text{ W m}{}^{-1} \text{ K}{}^{-1}$ ; $I = 6 \times 10^{-4} \text{ to}$
	$1.5 \times 10^{-3} \text{ mol dm}^{-3}$ ;
	$\epsilon_{\rm r} = 80$ for water;
	$\phi_{\text{exp}} = 0.003; d_{\text{p}} = 35 \text{ nm}; k_0{}^{b} = 1.34; D_{\text{f}}{}^{c} = 1.6;$
	$K_{\rm s}^{\ d} = 69 \text{ W m}^{-1} \text{ K}^{-1}$ ; for CuO aggregates
model	C  in eq  1 = 2.0; a  in eq  2 = 0.54
physical	$\epsilon_0 = 8.854 \times 10^{-12} \text{ C V}^{-1} \text{ m}^{-1}; F = 96 \ 485 \ \text{C mol}^{-1};$
constants	$R_0 = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

<sup>*a*</sup> Mean value of thermal conductivities of base fluids with no particles measured at different pHs. <sup>*b*</sup> From ref 33. <sup>*c*</sup> Kept constant throughout the model evaluation unless otherwise noted. <sup>*d*</sup> From ref 28.



**Figure 6.** Predictions of the effects of volume fraction for various CuO–water nanofluids at pH = 7. The solid line shows the prediction for the data of Li and Peterson;<sup>15</sup> the dashed line predicts the data of Eastman et al.,<sup>16</sup> and the dotted line predicts the data of Das et al.<sup>10</sup> Note that the prediction for Das et al. is made with  $D_f = 2.1$ , while  $D_f = 1.6$  is used for the others.

The author attempts to answer the question using the present model. As a surprising result, the model predicts reasonably well all of the trends. This implies that the disagreement in experiments reflects differences in some parameters. Li and Petterson's nanofluid contains smaller primary particles at higher temperature than that of Eastman et al. The former nanofluid has the advantage over the latter in effects of size and temperature, which explains the difference in the  $K_{\rm eff}$  value at small  $\phi_{\rm exp}$ . The values for  $D_{\rm f}$ and properties of the EDL used for the prediction in Figure 5 are consistently taken for the production of Figure 6. At the same  $D_{\rm f}$  (=1.6) and  $R_{\rm h}$ , if the radius of primary particles *a* decreases, the number density n of aggregates increases according to eqs 3 and 11, which corresponds to Li and Petterson's case. Also, regarding the fact that the EDL in the case of smaller a is relatively thicker, the effective volume fraction  $\phi_3$  increases more rapidly than in the nanofluid of Eastman et al. and eventually reaches unity at  $\phi_{exp} = 0.03$ . That is why  $K_{eff}$  of Li and Petterson's nanofluid does not increase further at  $\phi_{exp} > 0.03$ .

For the data from Das et al. in the figure, the model for  $D_{\rm f}$  = 1.6 overestimates their data (not shown here). Rather,  $D_{\rm f}$  = 2.1 was found for the best fit, which might be justified by looking at the TEM image of their aggregates.<sup>10</sup> The aggregates have a much more compact structure than our aggregates have (see Figure 1 in ref 14), reflecting higher  $D_{\rm f}$  for their aggregates. One may notice that their data are best fitted with  $D_{\rm f}$  = 2.1 at 21 °C and with  $D_{\rm f}$  = 1.6 at higher *T* (see Figure 5). As temperature increases, the surface charge increases along with electrostatic repulsion (see Table 3), which forces primary particles to repel each other, leading to a decrease in  $D_{\rm f}$ . This might be consistent with the sudden increase of the  $K_{\rm eff}$  in Figure 5 when *T* increases from 21 to 25 °C. Does the  $D_{\rm f}$  change in reality? Unfortunately, the author is not in the position to explain what happened exactly

 $3.28 \times 10^{5}$ 

 $4.99 \times 10^{5}$ 

60

70

Table 2. Parameters	s Used for	Prediction	of the	pH De	pendence	at 25	°C
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		Table 2. Paran	leters Used for	Prediction of the	ph Dependence	at 25°C	
pH	R <sub>h</sub> /nm	$\kappa^{-1}/nm$	$\Gamma_{ion}/mol \ m^{-1}$	$\sigma_{\rm d}/\sigma_{\rm d}/\sigma_{d$	$C m^{-2}$	$\phi_1$	$K_l\!/W\ m^{-1}K^{-1}$
3.0	88.4	7.94	$5.65 \times 10^{-1}$	7 5.44	$10^{-2}$	$1.44 \times 10^{-1}$	19.92
5.6	93.9	13.72	$5.29 \times 10^{-1}$	8 5.08	$3 \times 10^{-3}$	$5.91 \times 10^{-2}$	1.73
7.0	111.0	13.75	$1.40 \times 10^{-1}$	8 1.29	$0 \times 10^{-3}$	$5.88 \times 10^{-2}$	0.87
8.0	128.2	13.74	$4.43 \times 10^{-1}$	9 1.70	$0 \times 10^{-4}$	$5.89 \times 10^{-2}$	0.66
10.0	132.3	12.55	$2.56 \times 10^{-1}$	8 -2.44	$10^{-3}$	$6.93 \times 10^{-2}$	1.14
11.0	121.7	7.94	$8.30 \times 10^{-1}$	8 -7.99	$0 \times 10^{-3}$	$1.44 \times 10^{-1}$	3.04
	Та	ble 3. Parameters	Used for Predic	tion of the Temp	erature Depende	nce at pH = 7	
T/°C	$K_{\rm p}/{ m M}^{-1}$	$K_{ m d}/{ m M}$	$k^{-1}/nm$	$\Gamma_{ion}\!/mol~m^{-2}$	$s_{\rm d}/{\rm C}~{\rm m}^{-2}$	$f_1$	$K_{\rm l}/{\rm W}~{\rm m}^{-1}~{\rm K}^{-1}$
25	$6.00 \times 10^{4}$	$2.00 \times 10^{-12}$	13.75	$1.40 \times 10^{-8}$	$1.29 \times 10^{-3}$	$5.88 \times 10^{-2}$	0.87
30	$7.83 \times 10^{4}$	$2.61 \times 10^{-12}$	13.86	$1.63 \times 10^{-8}$	$1.49 \times 10^{-3}$	$5.79 \times 10^{-2}$	0.91
40	$1.30 \times 10^{5}$	$4.34 \times 10^{-12}$	14.09	$2.16 \times 10^{-8}$	$1.90 \times 10^{-3}$	$5.62 \times 10^{-2}$	1.01
50	$2.09 \times 10^{5}$	$6.98 \times 10^{-12}$	14.31	$2.81 \times 10^{-9}$	$2.34 \times 10^{-3}$	$5.46 \times 10^{-2}$	1.12

 $3.65 \times 10^{-8}$ 

 $5.55 \times 10^{-8}$ 

14.53

14.75

 $2.80 \times 10^{-3}$ 

 $2.84 \times 10^{-3}$ 

between 21 and 25 °C, because the necessary data, in particular, the fractal information, is not available.

 $1.09\times10^{-11}$ 

 $1.66 \times 10^{-11}$ 

Finally, the influence of primary particle size is investigated. Regarding the issue of  $D_{\rm f}$ , low  $D_{\rm f}$  (=1.6) is used for the predictions for the nanofluids of Li and Peterson<sup>15</sup> and Eastman et al.,<sup>16</sup> while high  $D_{\rm f}$  (=2.1) is used for those of Das et al.<sup>10</sup> and Lee et al.<sup>11</sup> Since all the literature does not report  $R_{\rm h}$ , the value at pH = 7 in Table 2 is used for the predictions in Figure 7. The figure shows that the model predicts quite well the two distinct trends of experimental data. For the same *T* and pH, the thermal conductivity and thickness of the interfacial layer is not varied.

As primary particles get smaller in this case, relatively thicker EDL and larger  $N_p$  increase  $\phi_2 (\propto 1/a^{\text{Df}}$ ; see eqs 3 and 9) much faster than they decrease  $\phi_1$  and  $K_{c^1}$  (refer to eqs 6 and 7 and Figure 3). This results in a slight increase of  $K_{c^2}$  and greater increase in  $\phi_3 (\propto 1/a^{3-\text{Df}}$ ; see eqs 3, 11, 12), which is a primary reason for enhancing the  $K_{\rm eff}$ . This explains the rapid increase of the  $K_{\text{eff}}$  in the region of a > 10 nm at  $D_{\text{f}} = 1.6$  in Figure 7. As a decreases further from 10 nm, the  $\phi_2$  reaches unity eventually. Nearly at the same time, the  $\phi_3$  reaches unity too. In the calculation, once  $\phi_2$  or  $\phi_3$  reaches unity, a further increase in each term is not considered. However,  $K_{c^1}$  and  $K_{c^2}$  keep decreasing slightly due to a decrease of the actual solid fraction in an equivalent sphere with a radius of R + t. That is why there exists a maximum in  $K_{\rm eff}$  for low  $D_{\rm f}$ . This behavior with respect to the primary particle size was also observed experimentally by Xie et al.<sup>4</sup> for an Al<sub>2</sub>O<sub>3</sub>-ethylene glycol nanofluid.

For the case of high  $D_{\rm f}$ , aggregates have a denser structure where the EDLs have more chance to be overlapped even for the same a. This means that the  $\phi_2$  increases to unity faster than for low  $D_{\rm f}$ . However, eq 5 shows that R for  $D_{\rm f} = 2.1$  is smaller than that for  $D_{\rm f} = 1.6$ , implying that  $\phi_3$  increases more slowly than for  $D_{\rm f} = 1.6$ . From this reason, the  $K_{\rm eff}$  is shown to increase with decreasing *a* over the whole size range. The predictions with two different fractal dimensions agree reasonably well with two groups of experimental data in Figures 6 and 7. It is likely that the fractal dimensions of the two categorized samples are not the same. Even though the present model resolves successfully the previous anomalies and contradictions in experiments, combining the concepts of fractal and EDL, the author would like to emphasize that the complete data sets such as surface charge states and fractal information should be reported for a better understanding of the abnormal behavior of nanofluids as well as proper evaluation of any theoretical models.

## 6. Discussion

A. Interpretation of the Conventional Interfacial Layer with Electrical Double Layer. The concept of liquid ordered (or structured) layering has been thought different from the EDL.37 Molecular ordering at surfaces results in structural and hydration forces. From the other side, EDL force produces electrostatic repulsion. One may have a question about the proximity of the interfacial layer to the EDL. Before discussing it, let me refer to very interesting experimental results<sup>38</sup> where an optical technique such as sum-frequency generation was used to probe liquid molecular orientation at a surface. The results showed that water molecular ordering was greatly pronounced at the SiO<sub>2</sub>/ water interface as pH increases far from the point of zero charge (PZC) of SiO<sub>2</sub>. Also, the paper presented that, at a neutral surface (pH = PZC) of SiO<sub>2</sub>, the hydration force induced the single layer ordering by hydrogen bonds, whereas, at a highly charged surface, the spectral intensity corresponding to icelike structure was increased by a factor of 7 from the value at pH = PZC, implying that the electrostatic force induced longer-range orientation of water molecules at the ionized surface. Moreover, it was observed that Na<sup>+</sup> ions existing in the liquid could magnify the influence of electrostatic force on the molecular ordering at an intermediate pH.

 $5.31 \times 10^{-2}$ 

 $5.16 \times 10^{-2}$ 

These experimental results seem to say that, if the surface is ionized, surface ionized sites are better able to orient polar water molecules and/or counterions (hydrated or not) by hydrogen bonding and/or ionic bonding up to the longer range. Such ion associations should result in a gradual decrease in the potential. This is very similar to the description of Figure 2 and obviously indicates that a new type of ordering can be possible by electrostatic force due to the existence of the EDL. I focused on the electrostatic interaction between particle surface and surrounding water containing ions, rather than between the particles. In this way, the interfacial layer may be interpreted to or approximated with the EDL.

**B.** The Validity of the Power-Law Relation for Thermal Conductivity of the EDL. The effective thermal conductivity  $K_{\text{eff}}$  and thermal conductivity of the EDL  $K_1$  can display different functional dependence versus  $\Gamma_{\text{ion}}\sigma_{d\kappa}$ . To determine the validity of the assumption in eq 2, I simulate the behaviors of the  $K_{c^1}$ ,  $K_{c^2}$ , and  $K_{\text{eff}}$  when independently varying  $K_1$  from 0.6 to 20 (highest value at pH = 3). For the purpose of this simulation, other parameters such as  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$ ,  $\kappa^{-1}$ , and R are fixed constant during the simulation with values at pH = 6. As a result, Figure 8 shows that  $K_{c^1}$  and  $K_{c^2}$  both have nearly linear dependences on  $K_1$ , while  $K_{\text{eff}}$  has a power dependence on  $K_1$ . Because  $K_{\text{eff}}$ 

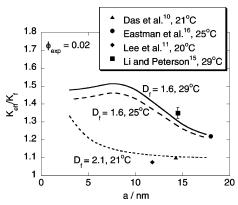
1.25

1.42

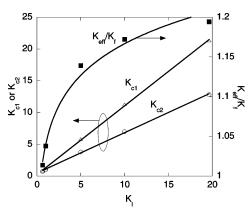
<sup>(36)</sup> Hiemstra, T.; Van Riemsdijk, W. H.; Bolt, G. H. J. Colloid Interface Sci.
1989, 133, 91–104 and 105–116.
(37) Israelachvili, J. N. Intermolecular and Surface Forces, 2nd ed.; Academic

Press: London, 1992.

<sup>(38)</sup> Du, Q.; Freysz, E.; Shen, Y. R. Phys. Rev. Lett. 1994, 72, 238-241.



**Figure 7.** Predictions of the effects of primary particle size for various CuO-water nanofluids at pH = 7 and  $\phi = 0.02$ . The solid line shows the prediction at  $D_f = 1.6$  and T = 29 °C, the dashed line predicts the size dependence at T = 25 °C, and the dotted line does at  $D_f = 2.1$  and T = 21 °C.

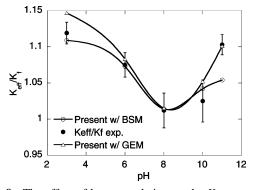


**Figure 8.** Functional dependences of  $K_{c^1}$ ,  $K_{c^2}$ , and  $K_{eff}/K_f$  on  $K_1$ ; solid lines for  $K_{c^1}$  and  $K_{c^2}$  represent linear-fitted lines, while the line for  $K_{eff}/K_f$  denotes a power-fitted line.

was displayed well with a power function of  $\Gamma_{ion}\sigma_{d\kappa}$  from experiment,<sup>14</sup> it is plausible to assume that  $K_1$  has also a power dependence on the  $\Gamma_{ion}\sigma_{d\kappa}$ , but the exponents for  $K_1$  and  $K_{eff}$  would be different.

C. Percolation Behavior of Heat through Aggregate Backbones. As the present model treats an agglomerate enclosed by a liquid sphere (see step 2) as an effective spherical sphere, the percolation should affect the prediction of the effective thermal conductivity  $K_c^2$  of the effective sphere. Therefore, the use of Bruggeman's effective medium theory (EMT) in step 2 has to be justified. Strictly, the EMT corresponds to Bruggeman's symmetric model (BSM), and its use should be restricted for effectively homogeneous and non-fractal media. For fractal and percolating media, the more general effective medium theory (GEM) of McLachlan et al.<sup>39,40</sup> that includes conductivity exponent t' and percolation threshold concentration  $f'_c$  should be applied.

According to McLachlan et al.,<sup>39</sup> the percolation thresholds (volume fraction of more conducting inclusions) were not varied much from 0.16 for various lattices. Also, McLachlan<sup>40</sup> suggested that the exponent was related to the threshold as  $t' = 3f'_c$  (see p 870 in ref 40). With  $f'_c$  and t',  $K_c^2$  at various pHs is estimated by the GEM. The GEM-generated  $K_c^2$  considering the percolation effect was obtained under the assumption of perfect contact between primaries (e.g., sharp prolate ellipsoids), so that  $K_c^2$ 



**Figure 9.** The effect of heat percolation on the  $K_{\text{eff}}$ . would be a upper limit of  $K_{c^2,\text{real}}$ , whereas  $K_{c^2}$  estimated from BSM in Step 2 would be a lower limit because the symmetric model does not consider the percolation effect.

By applying the  $K_{c^2,GEM}$  to the M–G model (eq 13), one may estimate the upper limit of the  $K_{eff}$ . Accordingly, the  $K_{c^2,BSM}$ presented in Figure 4 corresponds to lower limit. Figure 9 shows a very promising result that the experimental data are between the upper and lower limits. Note that the two limits get close each other in the region of 6 < pH < 10. A lower surface charge when pH close to PZC leads to a reduction in the contribution of the charge-induced interfacial layer ( $K_{c^1}$  closer to  $K_f$ ), so that the percolation effect becomes insignificant.

#### 7. Conclusions

In this study, the author employs the theory of the electrical double layer to investigate the unknown nature of the conventional interfacial layer. On the basis of this conceptual approach, a new fractal model has been developed and evaluated for various systematic parameters. The model was found to predict successfully experimental results for the parameters, i.e., pH, temperature, volume fraction, and primary particle size. Most of the presented data have not been understood by any preexisting models. The chemistry-based nature of the interfacial layer that is introduced in this study seems to play a key role in the control of the thermal conductivity of nanofluids.

Acknowledgment. The author gratefully acknowledges that this work was supported by Korea Research Foundation Grant funded by Korean Government (MOEHRD) (project no. R08-2003-000-10858-0).

### Appendix

In our previous work,<sup>14</sup> a theoretical approach, the so-called surface complexation model, to give the surface charge states was introduced in detail. To help understand the nature of the surface charging, a simplified model is presented here. The surface charging arisen when a particle is suspended in a polar liquid is described by<sup>14</sup>

$$MOH + H^{+} \xrightarrow{K_{p}} MOH_{2}^{+} \quad K_{p} = \exp\left(\frac{F\psi_{0}}{RT}\right) \frac{\Gamma(MOH_{2}^{+})}{a(H^{+})\Gamma(MOH)}$$
$$MOH \xrightarrow{K_{d}} MO^{-} + H^{+} \quad K_{d} = \exp\left(-\frac{F\psi_{0}}{RT}\right) \times \frac{\Gamma(MO^{-})a(H^{+})}{\Gamma(MOH)} (A1)$$

where M is a metal cation, i.e., Cu(II) in this case,  $K_p$  and  $K_d$  are equilibrium constants for protonation and deprotonation, respectively, *a* is activity in the bulk of the solution, and  $\psi_0$  is the surface potential. The equilibrium constants are not inde-

<sup>(39)</sup> McLachlan, D. S.; Blaszkiewicz, M.; Newnham, R. E. J. Am. Ceram. Soc. **1990**, 73, 2187–2203.

<sup>(40)</sup> McLachlan, D. S. J. Phys. C: Solid State Phys. 1987, 20, 865-877.

pendent of each other, but rather interrelated with the point of zero charge (PZC) of particles in water

$$PZC = \frac{1}{2} \log \left(\frac{K_{\rm p}}{K_{\rm d}}\right) \tag{A2}$$

The temperature dependence of the constant  $K_d$  is expressed by the van't Hoff equation, and Hiemstra et al.<sup>35</sup> suggested

$$\ln\left[\frac{K_{\rm d,T}}{K_{\rm d,T_0}}\right] = -\frac{\Delta H_0}{R}\left[\frac{1}{T} - \frac{1}{T_0}\right] \tag{A3}$$

The  $\Delta H_0$  of metal oxide ranges from 15 to 90 kJ/mol,<sup>35</sup> and 40 kJ/mol was taken for the estimation of the  $K_d$  in this study. In this way, when *T* increases from 25 to 70 °C, each of the  $K_p$  and  $K_d$  values increases by 1 order of magnitude, as seen in Table 3. At this moment, the charged cites, regardless of their polarities, can be associated further with counterions in the EDL as seen in Figure 2. In the previous work,<sup>14</sup> the surface complexation by the counterions was considered. But at the low concentration of electrolyte, the association effect is quite negligible<sup>22</sup> and therefore neglected in this study. A more simplified model to estimate the surface potential is thus readily derived as follows. The difference between the previous and present models is displayed in Figure 2.

Rearranging the terms in eq A1, the charged site densities  $(\Gamma(\text{MOH}_2^+) \text{ and } \Gamma(\text{MO}^-))$  are expressed as a function of uncharged site density  $\Gamma(\text{MOH})$ . The total sum of charged and uncharged site densities is equal to the total surface site density  $\Gamma_{\text{tot}}$  (ca. 5.9 × 10<sup>-6</sup> mol/m<sup>2</sup>).<sup>14</sup> Using this, one can easily derive the following equations

$$\Gamma(\text{MOH}) = \frac{\Gamma_{\text{tot}}}{1 + a_{\text{s}}(\text{H}^+)K_{\text{p}} + K_{\text{d}}/a_{\text{s}}(\text{H}^+)} \qquad (A4)$$

$$\Gamma(\text{MOH}_{2}^{+}) = \frac{\Gamma_{\text{tot}}a_{\text{s}}(\text{H}^{+})K_{\text{p}}}{1 + a_{\text{s}}(\text{H}^{+})K_{\text{p}} + K_{\text{s}}/a_{\text{s}}(\text{H}^{+})}$$
(A5)

$$\Gamma(\text{MO}^{-}) = \frac{\Gamma_{\text{tot}} K_{\text{d}} / a_{\text{s}}(\text{H}^{+})}{1 + a_{\text{s}}(\text{H}^{+}) K_{\text{p}} + K_{\text{d}} / a_{\text{s}}(\text{H}^{+})}$$
(A6)

where the activity of  $H^+$  at the surface is given by the Maxwell–Boltzman distribution<sup>22</sup>

$$a_{\rm s}({\rm H}^+) \equiv a({\rm H}^+) \exp\left(-\frac{F\psi_0}{RT}\right)$$
 (A7)

A sum of the charged site densities ( $\Gamma(MOH_2^+) + \Gamma(MO^-)$ ) is referred to as  $\Gamma_{ion}$ . Surface charge density  $\sigma_0$  is given by the definition

$$\sigma_{0} = F[\Gamma(\text{MOH}_{2}^{+}) - \Gamma(\text{MO}^{-})] = \frac{\Gamma_{\text{tot}}F}{1 + a_{\text{s}}(\text{H}^{+})K_{\text{p}} + K_{\text{d}}/a_{\text{s}}(\text{H}^{+})} [a_{\text{s}}(\text{H}^{+})K_{\text{p}} - K_{\text{d}}/a_{\text{s}}(\text{H}^{+})]$$
(A8)

With known values of  $K_p$  and  $K_d$  at various temperatures,  $\sigma_0$  is an only function of  $a_s(H^+)$  or a function of  $\psi_0$  and *T*. The  $\sigma_0$ (approximated to  $\sigma_s$  by the simplified model in Figure 2) is balanced with the charge in the diffuse layer as

$$\sigma_0 = -\sigma_d = -\frac{4FI}{\kappa} \sinh\left(\frac{F\psi_d}{2RT}\right) \approx \frac{4FI}{\kappa} \sinh\left(\frac{F\psi_0}{2RT}\right)$$
(A9)

By equating eq A8 to eq A9, one gets a nonlinear equation for  $\psi_0$  and *T*. At a certain *T*, numerical iteration for  $\psi_0$  can easily give  $\psi_0$  as a function of *T*. Once  $\psi_0$  is given, one can obtain all surface site densities and charge density  $\sigma_d$  as well (using eqs A4–6 and A9, respectively).

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